

SPM BULLETIN

ISSUE NUMBER 30: June 2010 CE

1. EDITOR'S NOTE

Due to shortage in free time for editing the *SPM Bulletin*, I am trying for a while a “slim” version containing mainly, when the papers are not in the core of the field of selection principles, titles and links. By clicking a link, you will get directly to the webpage of the paper, including the abstract and full paper. This will make it possible to post fewer issues and save time for the editor and the readers.

The new style make the table of contents less important, and we therefore do not include it anymore.

We apologize for not being able to keep up with the earlier format.

Boaz Tsaban, tsaban@math.biu.ac.il
<http://www.cs.biu.ac.il/~tsaban>

2. RESEARCH IN SPM

Note. The division between the present section and the next one is somewhat artificial, and cannot be precise.

2.1. Ideals which generalize (v^0) . We consider ideals $d^0(\mathcal{V})$ which are generalizations of the ideal (v^0) . We formulate counterparts of Hadamard's theorem. Then, adopting the base tree theorem and applying Kulpa-Szymański Theorem, we obtain $\text{cov}(d^0(\mathcal{V})) \leq \text{add}(d^0(\mathcal{V}))^+$.

<http://arxiv.org/abs/1001.5400>
Piotr Kalembe and Szymon Plewik

2.2. Preserving the Lindelöf property under forcing extensions. We investigate preservation of the Lindelöf property of topological spaces under forcing extensions. We give sufficient conditions for a forcing notion to preserve several strengthenings of the Lindelöf property, such as indestructible Lindelöf property, the Rothberger property and being a Lindelöf P -space.

<http://arxiv.org/abs/1002.0894>
Masaru Kada

2.3. Remarks on the preservation of topological covering properties under Cohen forcing. Iwasa investigated the preservation of various covering properties of topological spaces under Cohen forcing. By improving the argument in Iwasa's paper, we prove that the Rothberger property, the Menger property and selective screenability are also preserved under Cohen forcing and forcing with the measure algebra.

<http://arxiv.org/abs/1002.4419>

Masaru Kada

2.4. A unified theory of function spaces and hyperspaces: local properties. Many classically used function space structures (including the topology of pointwise convergence, the compact-open topology, the Isbell topology and the continuous convergence) are induced by a hyperspace structure counterpart. This scheme is used to study local properties of function space structures on $C(X, \mathbb{R})$, such as character, tightness, fan-tightness, strong fan-tightness, the Fréchet property and some of its variants. Under mild conditions, local properties of $C(X, \mathbb{R})$ at the zero function correspond to the same property of the associated hyperspace structure at X . The latter is often easy to characterize in terms of covering properties of X . This way, many classical results are recovered or refined, and new results are obtained. In particular, it is shown that tightness and character coincide for the continuous convergence on $C(X, \mathbb{R})$ and is equal to the Lindelöf degree of X . As a consequence, if X is consonant, the tightness of $C(X, \mathbb{R})$ for the compact-open topology is equal to the Lindelöf degree of X .

<http://arxiv.org/abs/1002.2883>

S. Dolecki and F. Mynard

2.5. Generalized Luzin sets. In this paper we investigate the notion of generalized (I, J) -Luzin set. This notion generalize the standard notion of Luzin set and Sierpinski set. We find set theoretical conditions which imply the existence of generalized (I, J) -Luzin set. We show how to construct large family of pairwise non-equivalent (I, J) -Luzin sets. We find a class of forcings which preserves the property of being (I, J) -Luzin set.

<http://arxiv.org/abs/1003.0714>

Robert Ralowski and Szymon Zeberski

2.6. Skeletal maps and I -favorable spaces. It is showed that the class of all compact Hausdorff and I -favorable spaces is adequate for the class of skeletal maps.

<http://arxiv.org/abs/1003.2308>

Andrzej Kucharski and Szymon Plewik

2.7. Hereditarily Hurewicz spaces and Arhangel'skii sheaf amalgamations. A classical theorem of Hurewicz characterizes spaces with the Hurewicz covering property as those having bounded continuous images in the Baire space. We give a

similar characterization for spaces X which have the Hurewicz property hereditarily. We proceed to consider the class of Arhangel'skii α_1 spaces, for which every sheaf at a point can be amalgamated in a natural way. Let $C_p(X)$ denote the space of continuous real-valued functions on X with the topology of pointwise convergence. Our main result is that $C_p(X)$ is an α_1 space if, and only if, each *Borel* (!) image of X in the Baire space is bounded. Using this characterization, we solve a variety of problems posed in the literature concerning spaces of continuous functions.

To appear in *Journal of the European Mathematical Society*.

<http://arxiv.org/abs/1004.0211>

Boaz Tsaban, Lyubomyr Zdomskyy

Note. Recently, Lev Bukovský and Jaroslav Šupina discovered an alternative proof of the main result of this paper. Their paper is expected to be available before long.

2.8. The relation of rapid ultrafilters and Q -points to van der Waerden ideal. We point out one of the differences between rapid ultrafilters and Q -points: Rapid ultrafilters may have empty intersection with van der Waerden ideal, whereas every Q -point has a non-empty intersection with van der Waerden ideal. Assuming Martin's axiom for countable posets we also construct a W -ultrafilter which is not a Q -point.

<http://arxiv.org/abs/1004.1879>

Jana Flašková

2.9. External characterization of I -favorable spaces. We provide both a spectral and internal characterizations of arbitrary I -favorable spaces. As a corollary we establish that any perfect image of an openly generated space is I -favorable. In particular, any image of compact κ -metrizable space is I -favorable. The last statement is a generalization of a result due to P. Daniels, K. Kunen and H. Zhou. We also generalize a result of Berezničii by proving that every I -favorable subspace of extremally disconnected space is extremally disconnected.

<http://arxiv.org/abs/1005.0074>

Vesko Valov

2.10. The character of topological groups: Shelah's pcf theory and Pontryagin-van Kampen duality. The minimal cardinality of a base at the identity in a topological group G , denoted $\chi(G)$, is one of the major invariants of G . A celebrated 1936 result of Birkhoff and (independently) Kakutani asserts that G is metrizable if, and only if, $\chi(G)$ is countable.

We consider the case where G is the *dual group* of a metrizable group. Using Pontryagin-van Kampen duality and pcf theory, we show that also in this case, $\chi(G)$ is well behaved, and that it is determined by the density and the local density of the base, metrizable group.

We apply our result to compute the character of free abelian topological groups, extending a number of results of Nickolas and Tkachenko.

This phenomenon is also reformulated in an inner language, not referring to duality theory. Here, the compact subsets of quotients by compact subgroups of G determine its character.

For G dual to a metrizable group, $\chi(G)$ is especially well behaved in the absence of large cardinals. On the other hand, when large cardinals are available, some freedom is demonstrated using Cohen's "forcing" method, answering a question of Bonanzinga and Matveev.

Please comment!. This is a preliminary version, and none of the authors is an expert in *all* topics dealt with in the paper. We are very interested in receiving comments and suggestions from experts in topological groups, duality theory, and set theory (forcing, large cardinals, pcf theory).

<http://arxiv.org/abs/1005.0577>

Cristina Chis, M. Vincenta Ferrer, Salvador Hernandez, Boaz Tsaban

2.11. Sequential properties of function spaces with the compact-open topology. Let M be the countably infinite metric fan. We show that $C_k(M, 2)$ is sequential and contains a closed copy of Arens space S_2 . It follows that if X is metrizable but not locally compact, then $C_k(X)$ contains a closed copy of S_2 , and hence does not have the property AP.

We also show that, for any zero-dimensional Polish space X , $C_k(X, 2)$ is sequential if and only if X is either locally compact or the derived set X' is compact. In the case that X is a non-locally compact Polish space whose derived set is compact, we show that all spaces $C_k(X, 2)$ are homeomorphic, having the topology determined by an increasing sequence of Cantor subspaces, the n th one nowhere dense in the $(n+1)$ st.

<http://arxiv.org/abs/1005.5542>

Gary Gruenhage, Boaz Tsaban, and Lyubomyr Zdomskyy

2.12. Pcf theory and cardinal invariants of the reals. The additivity spectrum $ADD(I)$ of an ideal I is the set of all regular cardinals κ such that there is an increasing chain $\{A_\alpha : \alpha < \kappa\}$ in the ideal I such that the union of the chain is not in I . We investigate which set A of regular cardinals can be the additivity spectrum of certain ideals.

Assume that I is the sigma-ideal generated by the compact subsets of the Baire space or the ideal of null sets. For countable sets we give a full characterization of the additivity spectrum of I : a non-empty countable set A of uncountable regular cardinals can be $ADD(I)$ in some c.c.c generic extension iff $A = \text{pcf}(A)$.

<http://arxiv.org/abs/1006.1808>

Lajos Soukup

3. RELATED RESEARCH

3.1. Getting more colors.

<http://arxiv.org/abs/0912.5366>

Todd Eisworth

3.2. A coloring theorem for sucesors of singular cardinal.

<http://arxiv.org/abs/1001.0194>

Todd Eisworth

3.3. Nonmeasurability in Banach spaces.

<http://arxiv.org/abs/1001.0073>

Robert Ralowski

3.4. Remarks on nonmeasurable unions of big point families.

<http://arxiv.org/abs/1001.0549>

Robert Ralowski

3.5. Zariski topologies on groups.

<http://arxiv.org/abs/1001.0601>

Taras Banach and Igor Protasov

3.6. GO-spaces and Noetherian spectra.

<http://arxiv.org/abs/1001.0596>

Authors: David Milovich

3.7. Understanding Preservation Theorems, II.

<http://arxiv.org/abs/1001.0922>

Chaz Schlindwein

3.8. Sequential order under CH.

<http://arxiv.org/abs/1001.0908>

Chiara Baldovino

3.9. Transfinite Approximation of Hindman's Theorem.

<http://arxiv.org/abs/1001.1175>

Mathias Beiglböck and Henry Towsner

3.10. Diamond, GCH and weak square.

<http://www.ams.org/journal-getitem?pii=S0002-9939-10-10192-0>

Martin Zeman

3.11. Forcing properties of ideals of closed sets.

<http://arxiv.org/abs/1001.2819>

Marcin Sabok, Jindrich Zapletal

3.12. **Precompact noncompact reflexive abelian groups.**

<http://arxiv.org/abs/1001.4895>

S. Ardanza-Trevijano, M. J. Chasco, X. Domínguez, M. G. Tkachenko

3.13. **Multiple gaps.**

<http://arxiv.org/abs/1001.4888>

Antonio Avilés, Stevo Todorčević

3.14. **A Combinatorial Proof of the Dense Hindman Theorem.**

<http://arxiv.org/abs/1002.0347>

Henry Towsner

3.15. **Common idempotents in compact left topological left semirings.**

<http://arxiv.org/abs/1002.1599>

Denis I. Saveliev

3.16. **Diamonds.**

<http://www.ams.org/journal-getitem?pii=S0002-9939-10-10254-8>

Saharon Shelah

3.17. **More on cardinal invariants of analytic P-ideals.**

<http://arxiv.org/abs/1002.2192>

Barnabás Farkas, Lajos Soukup

3.18. **When is the Isbell topology a group topology?**

<http://arxiv.org/abs/1002.2886>

S. Dolecki and F. Mynard

3.19. **Group topologies coarser than the Isbell topology.**

<http://arxiv.org/abs/1002.3089>

S. Dolecki, F. Jordan and F. Mynard

3.20. **Relations that preserve compact filters.**

<http://arxiv.org/abs/1002.3120>

F. Mynard

3.21. **Products of compact filters and applications to classical product theorems.**

<http://arxiv.org/abs/1002.3122>

F. Mynard

3.22. **New book: Structural Ramsey theory of metric spaces and topological dynamics of isometry groups.**

<http://www.ams.org/journal-getitem?pii=S0065-9266-10-00586-7>

L. Nguyen Van The

- 3.23. **A new Lindelof topological group.**
<http://arxiv.org/abs/1002.4508>
Dusan Repovs, Lyubomyr Zdomskyy
- 3.24. **A Model Theoretic Proof of Szemerédi's Theorem.**
<http://arxiv.org/abs/1002.4456>
Henry Towsner
- 3.25. **Nondiscrete P-Groups Can be Reflexive.**
<http://arxiv.org/abs/1002.4730>
Jorge Galindo, Luis Recoder-Nuñez and Mikhail Tkachenko
- 3.26. **Completely nonmeasurable unions.**
<http://arxiv.org/abs/1003.0918>
Robert Ralowski and Szymon Zeberski
- 3.27. **The Stationary Set Splitting Game.**
<http://arxiv.org/abs/1003.2425>
Paul Larson and Saharon Shelah
- 3.28. **Universally measurable sets in generic extensions.**
<http://arxiv.org/abs/1003.2479>
Paul Larson, Itay Neeman, and Saharon Shelah
- 3.29. **Dense families of countable sets below \mathfrak{c} .**
<http://arxiv.org/abs/1003.2496>
Lajos Soukup
- 3.30. **A note on Noetherian type of spaces.**
<http://arxiv.org/abs/1003.3189>
Lajos Soukup
- 3.31. **Uniformizing ladder system colorings and the rectangle refining.**
<http://www.ams.org/journal-getitem?pii=S0002-9939-10-10330-X>
Teruyuki Yorioka
- 3.32. **Sections, Selections and Prohorov's Theorem.**
<http://arxiv.org/abs/1003.4024>
V. Gutev and V. Valov
- 3.33. **Minimal functions on the random graph and the product Ramsey theorem.**
<http://arxiv.org/abs/1003.4030>
Manuel Bodirsky, Michael Pinsker

- 3.34. Complexity of Ramsey null sets.
<http://arxiv.org/abs/1003.5983>
Marcin Sabok
- 3.35. Conflict free colorings of (strongly) almost disjoint set-systems.
<http://arxiv.org/abs/1004.0181>
András Hajnal, István Juhász, Lajos Soukup, Zoltán Szentmiklóssy
- 3.36. Fine asymptotic densities for sets of natural numbers.
<http://www.ams.org/journal-getitem?pii=S0002-9939-10-10351-7>
Mauro Di Nasso
- 3.37. The character spectrum of $\beta\mathbb{N}$.
<http://arxiv.org/abs/1004.2083>
Saharon Shelah
- 3.38. Baire class one colorings and a dichotomy for countable unions of F_σ rectangles.
<http://arxiv.org/abs/1004.2172>
Dominique Lecomte
- 3.39. Linear ROD subsets of Borel partial orders are countably cofinal in the Solovay model.
<http://arxiv.org/abs/1004.5542>
Vladimir Kanovei
- 3.40. The Markov-Zariski topology of an abelian group.
<http://arxiv.org/abs/1005.1149>
Dikran Dikranjan, Dmitri Shakhmatov
- 3.41. Some more problems about orderings of ultrafilters.
<http://arxiv.org/abs/1005.2590>
Paolo Lipparini
- 3.42. Thin-very tall compact scattered spaces which are hereditarily separable.
<http://arxiv.org/abs/1005.3528>
Christina Brech and Piotr Koszmider
- 3.43. The extender algebra and Σ_1^2 -absoluteness.
<http://arxiv.org/abs/1005.4193>
Ilijas Farah
- 3.44. Compactness in Banach space theory - selected problems.
[http://arxiv.org/abs/1005.4303\(*cross-listing*\)](http://arxiv.org/abs/1005.4303(*cross-listing*))
Antonio Avilés and Ondřej F.K. Kalenda

3.45. Linear ROD subsets of Borel partial orders are countably cofinal in Solovay's model.

<http://arxiv.org/abs/1005.5534>

Vladimir Kanovei

3.46. Wide scattered spaces and morasses.

<http://arxiv.org/abs/1006.1720>

Lajos Soukup

4. UNSOLVED PROBLEMS FROM EARLIER ISSUES

Issue 1. Is $\left(\frac{\Omega}{\Gamma}\right) = \left(\frac{\Omega}{\Gamma}\right)$?

Issue 2. Is $U_{\text{fin}}(\mathcal{O}, \Omega) = S_{\text{fin}}(\Gamma, \Omega)$? And if not, does $U_{\text{fin}}(\mathcal{O}, \Gamma)$ imply $S_{\text{fin}}(\Gamma, \Omega)$?

Issue 4. Does $S_1(\Omega, \Gamma)$ imply $U_{\text{fin}}(\Gamma, \Gamma)$?

Issue 5. Is $\mathfrak{p} = \mathfrak{p}^*$? (See the definition of \mathfrak{p}^* in that issue.)

Issue 6. Does there exist (in ZFC) an uncountable set satisfying $S_{\text{fin}}(\mathcal{B}, \mathcal{B})$?

Issue 8. Does $X \notin \text{NON}(\mathcal{M})$ and $Y \notin \text{D}$ imply that $X \cup Y \notin \text{COF}(\mathcal{M})$?

Issue 9 (CH). Is $\text{Split}(\Lambda, \Lambda)$ preserved under finite unions?

Issue 10. Is $\text{cov}(\mathcal{M}) = \mathfrak{od}$? (See the definition of \mathfrak{od} in that issue.)

Issue 12. Could there be a Baire metric space M of weight \aleph_1 and a partition \mathcal{U} of M into \aleph_1 meager sets where for each $\mathcal{U}' \subset \mathcal{U}$, $\bigcup \mathcal{U}'$ has the Baire property in M ?

Issue 14. Does there exist (in ZFC) a set of reals X of cardinality \mathfrak{d} such that all finite powers of X have Menger's property $S_{\text{fin}}(\mathcal{O}, \mathcal{O})$?

Issue 15. Can a Borel non- σ -compact group be generated by a Hurewicz subspace?

Issue 16 (MA). Is there $X \subseteq \mathbb{R}$ of cardinality continuum, satisfying $S_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma)$?

Issue 17 (CH). Is there a totally imperfect X satisfying $U_{\text{fin}}(\mathcal{O}, \Gamma)$ that can be mapped continuously onto $\{0, 1\}^{\mathbb{N}}$?

Issue 18 (CH). Is there a Hurewicz X such that X^2 is Menger but not Hurewicz?

Issue 19. Does the Pytkeev property of $C_p(X)$ imply that X has Menger's property?

Issue 20. Does every hereditarily Hurewicz space satisfy $S_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma)$?

Issue 21 (CH). Is there a Rothberger-bounded $G \leq \mathbb{Z}^{\mathbb{N}}$ such that G^2 is not Menger-bounded?

Issue 22. Let \mathcal{W} be the van der Waerden ideal. Are \mathcal{W} -ultrafilters closed under products?

Issue 23. Is the δ -property equivalent to the γ -property $\left(\frac{\Omega}{\Gamma}\right)$?

Previous issues. The previous issues of this bulletin are available online at <http://front.math.ucdavis.edu/search?&t=%22SPM+Bulletin%22>

Contributions. Announcements, discussions, and open problems should be emailed to tsaban@math.biu.ac.il

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